Paging and Packing with Possibly Precise Predictions

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MAPSP 22

Paging and Packing with Possibly Precise Predictions

Online algorithms with advice (oracle)
Online algorithms with predictions (e.g. ML)

Upper & lower bound techniques

Running example: Paging
Vertex Cover, Knapsack, Bin Packing

Running example :

Paging

Cache with room for k pages

Input : Sequence of page requests

Requested page, p, not in cache : fault

→ Evict a page from the cache to make room for p

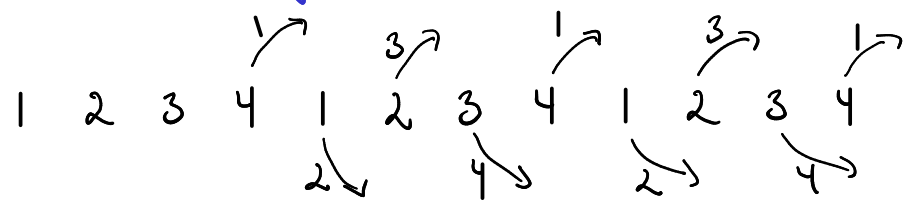
Objective : Minimize #faults

Online problem :

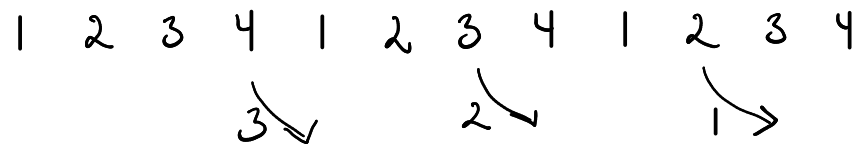
$I = r_1, r_2, r_3, \ldots, r_n$ → Online algorithm → $Y = y_1, y_2, y_3, \ldots, y_n$

Ex: $k = 3$

Least Recently Used (LRU)



Longest Forward Distance (LFD) (optimal offline alg.)



$$\frac{LRU}{LFD} = 3 = k$$

Competitive Ratio

If $\exists b : \forall I : \; ALG(I) \leq c \cdot OPT + b$
then ALG is c-competitive
The competitive ratio of ALG is
$CR(ALG) = \inf \{ c \mid ALG \text{ is } c\text{-competitive} \}$

Game between online player and malicious adversary

Adversary $\xrightarrow{I = r_1, r_2, r_3, \ldots, r_n}$ Online algorithm $\xrightarrow{Y = y_1, y_2, y_3, \ldots, y_n}$

Competitive Ratio

If $\exists b : \forall I : \; ALG(I) \leq c \cdot OPT + b$
then ALG is c-competitive

The competitive ratio of ALG is
$CR(ALG) = \inf \{ c \mid ALG \text{ is } c\text{-competitive} \}$

[ST85]:

For any deterministic paging alg. ALG, $CR(ALG) \geq k$

Proof:
Adversary Strategy:
  Use k+1 pages
  Always req. page ∉ ALG's cache

ALG faults on each req.

LFD faults on ≤ every k^{th} req.

Thus, $\frac{ALG}{LFD} \geq k$ □

[ST85]:
Sleator, Tarjan
Amortized Efficiency of List Update and Paging Rules
CACM 85

Competitive Ratio

If $\exists b : \forall I : E[ALG(I)] \leq c \cdot OPT + b$
then ALG is c-competitive

The competitive ratio of ALG is
$CR(ALG) = \inf \{ c \mid ALG \text{ is } c\text{-competitive} \}$

[FKLMSY 91]:

For any randomized paging alg. ALG, $CR(ALG) \geq H_k \approx \ln k$

[FKLMSY 91]:
Fiat, Karp, Luby, McGeoch, Sleator, Young
Competitive Paging Algorithms
J. Algorithms 91

[FKLMSY91]:

For any randomized paging alg. ALG, $CR(ALG) \geq H_k \approx \ln k$

Proof: (By Yao's Principle)

Adversary strategy:

Use $k+1$ pages

Choose page to be requested uniformly at random

ALG faults with prob. $\frac{1}{k+1}$

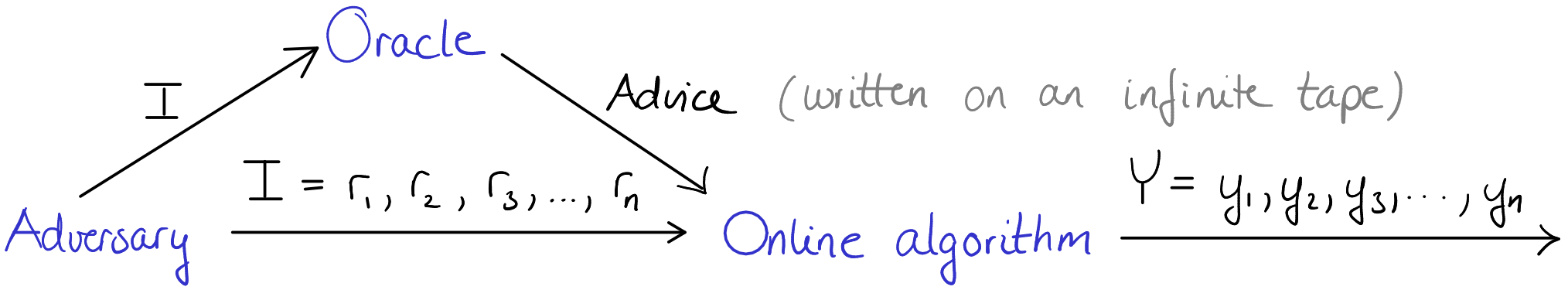
LFD faults once for each k distinct pages $\Rightarrow$

In expectation, LFD faults once for each $(k+1)H_{k+1} - 1 = (k+1)H_k$

Coupon Collector: $\underbrace{1\ 2\ 3\ 4 \cdots k}_{(k+1)H_{k+1}}\ \underbrace{k+1\ 1'\ 2'\ 3' \cdots k'}_{(k+1)H_{k+1}}$ □

$$\left(H_k = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k} \approx \ln k\right)$$

Online Algorithms with Advice



The advice complexity, $b_A(n)$, of an algorithm, A, is the max #advice bits read by A on any I of length n

The advice complexity, $b_P(n,c)$, of a problem, P, is the #advice bits necessary and sufficient to be c-competitive (as a function of input length), i.e.,

$$b_P(n,c) = \inf_{\text{c-comp. } A} \left\{ \max_{|I|=n} \{ b_A(n) \} \right\}$$

Advice Complexity

How much extra information does it take to
- be (offline) optimal ?
- beat best online algorithm ?

Algorithms may be stepping stones to
- semi-online algorithms
- parallel solutions
- algorithms with predictions

Lower bounds give impossibility results for
- semi-online algorithms
- parallel solutions
- randomized online algorithms (sometimes)

CR = 1:

First (trivial) idea: Encode OPT

For each fault, state the page that OPT would evict.
$\Rightarrow$ $b(n,1) \leq n \log k$

Next idea: Now-or-later [DKP 09]

For each request, indicate whether the page requested would stay in OPT's cache until its next request.
$\Rightarrow$ $b(n,1) \leq n$

Ex: $k = 3$

I = 1 2 3 4 2 5 1 2 3 5

ϕ = 0 0 1 1 0 0 ...

1: safe to evict
0: stay in cache

Note: $OPT = |\phi|_1$

[DKP 09]: Dobrev, Královič, Pardubská
Measuring the problem relevant information in input
RAIRO – Theoretical Informatics and Applications 2009

CR > 1 :

First idea: Use advice only for „easy" sequences [BKKKM 17]

OBS: For any alg., A, and any input, I, $OPT(I) \geq \frac{n}{c} \Rightarrow \frac{A(I)}{OPT(I)} \leq c$

Hence, a c-competitive algorithm needs advice, only if $|\phi|_1 < n/c$.

Thus, $b(n,c) \leq \log(\#\text{advice strings with} < \frac{n}{c}\ 1\text{'s})$

$= O(\frac{\log c}{c} n)$, for $c > 1$.

[BKKKM 17]: Böckenhauer, Komm, Královič, Královič, Mömke
Online algorithms with advice: The tape model
Information and Computation 2017

Note: For inputs with $|\phi|_1 < \frac{n}{c}$, the algorithm is optimal.
In this case, we can afford $(c-1)|\phi|_1$ flips $0 \to 1$

Next idea: Covering designs

Construct new advice string ϕ' „covering" all 1's in ϕ:

$$\phi \xrightarrow{(c-1)|\phi|_1 \text{ flips } 0\to 1} \phi'$$

Ex: $c=2$

ϕ: 0 0 | | 0 0 | 0 | 0 0 0 | 0 0 0 0

ϕ': 0 | | | | | | | 0 0 | | 0 0 0 0

The resulting algorithm is

- well-defined: there is always a page that is safe to evict, since $\phi_i = 1 \Rightarrow \phi'_i = 1$
- c-competitive: $|\phi'|_1 \leq c \cdot |\phi|_1$ and each 1 gives ≤ 1 fault

Since it is not important which 0's are flipped, we can use covering designs...

Covering design

For any set S, $|S| = n$, an (n,k,ℓ)-covering design, $\ell \le k \le n$, is a set $\mathcal{G}_{k,\ell}$ of k-subsets of S s.t. each ℓ-subset of S is contained in (covered by) one of the sets in $\mathcal{G}_{k,\ell}$.

Ex: A $(4,3,2)$-covering design

$S = \{1,2,3,4\}$

$\mathcal{G}_{3,2} = \{\{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$

$\{1,2\}$ $\{1,4\}$ $\{3,4\}$
$\{1,3\}$ $\{2,4\}$
$\{2,3\}$

Indicating one of the three 3-subsets in $\mathcal{G}_{3,2}$ takes $\lceil \log |\mathcal{G}_{3,2}| \rceil = 2$ bits.

For an $(n, c\cdot|\Phi|_1, |\Phi|_1)$-covering design $\mathcal{G}$ give n, $|\Phi|_1$, and the index of Φ' in $\mathcal{G}$ as advice.

Use an $(n, c|\phi|_1, |\phi|_1)$ - covering design $\mathcal{G}_{c|\phi|_1, |\phi|_1}$ to indicate a string ϕ' s.t.

- $\phi_i = 1 \Rightarrow \phi'_i = 1$
- $|\phi'|_1 \leq c \cdot |\phi|_1$

Give $\underbrace{n, |\phi|_1}_{O(\log n)}$, and the $\underbrace{\text{index of } \phi' \text{ in } \mathcal{G}_{c|\phi|_1, |\phi|_1}}_{\leq \frac{n}{c} + O(\log n), \text{ since}}$ as advice.

$$\max_{1 \leq |\phi|_1 \leq \frac{n}{c}} \log \left| \mathcal{G}_{c|\phi|_1, |\phi|_1} \right| = \underbrace{\log \left(1 + \frac{(c-1)^{c-1}}{c^c} \right) \cdot n}_{\frac{n}{2c} \leq B(n,c) \leq \frac{n}{c}} + O(\log n)$$

$\therefore b(n,c) \leq \frac{n}{c} + O(\log n)$

Covering designs can also be used for accept/reject problems, e.g.,

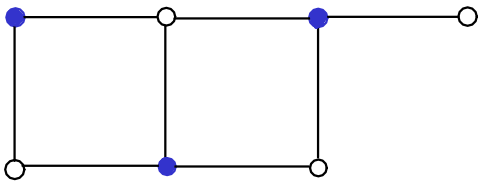
- Vertex Cover (VC): Any superset of a VC is a VC

VC

Input: $v_1 v_2 \cdots v_n$
With v_i, edges to $v_1, \ldots, v_{i-1}$ are revealed

Output: Set of vertices touching all edges

Ex:



- Independent Set (IS): Any subset of an IS is an IS

Use a $(n, n - \frac{|\phi|_1}{c}, |\phi|_0)$-covering design

$b(n,c) = B(n,c) + O(\log n) \leq \frac{n}{c} + O(\log n)$

VC and IS belong to

AOC (Asymmetric Online Covering):

Minimization problems:

For any feasible solution, X, cost(X) = |X|

Any superset of an optimal solution is a feasible solution

Maximization problems:

For any feasible solution, X, gain(X) = |X|

Any subset of an optimal solution is a feasible solution

Examples:

IS, VC, Dom. Set, Set Cover, Disj. Path Alloc., Cycle Finding

Knapsack (unit profit), Matching (edge arrival)

A problem $P \in AOC$ is AOC-complete, if

$b_p(n,c) \geq B(n,c) - O(\log n)$

A problem $P \in AOC$ is AOC-complete, if

$$b_P(n,c) \geq B(n,c) - O(\log n)$$

Completeness can be proven via advice preserving reductions.
from, e.g., Asymmetric String Guessing (ASG) [BFKM 16]:

ASG (with known history)

Input: $x_1 x_2 \cdots x_n$, $x_i \in \{0,1\}$
On request i, the alg. must guess x_i
(After the guess, x_i is revealed.)

Cost:

x_i \ y_i	0	1
0	0	1
1	∞	1

$$b(n,c) = B(n,c) \pm O(\log n)$$

Recall:

$$\frac{n}{2c} \leq B(n,c) \leq \frac{n}{c}$$

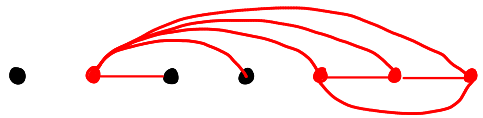
[BFKM 16]: Boyar, Favrholdt, Kudahl, Mikkelsen
A class of hard online problems
Theory of Computing Systems 2016

Ex: Reduction ASG → VC

A_{VC}: c-comp. VC-alg. with advice string ϕ_{VC}
A_{ASG}: c-comp. ASG-alg using A_{VC} and ϕ_{VC}

For any input X, A_{ASG} builds a graph G_X:

$X = 0\ 1\ 0\ 0\ 1\ 1\ 1$



●: V_0
●: V_1

V_1 induces a clique $\Rightarrow$ A_{VC} rejects ≤ 1 vertex from V_1

$OPT \leq |V_1| = |X|_1 \Rightarrow A_{VC}$ accepts $\leq c \cdot |X|_1$ vertices in total

Hence, $|\phi_{VC}| + O(\log n)$ bits suffice to
- guess all 1's correctly
- return $\leq c|X|_1$ 1's

Hence, $b_{VC}(n,c) \geq b_{ASG}(n,c) - O(\log n) \geq \frac{n}{2c} - O(\log n)$,

i.e., for VC, covering designs are optimal, up to an additive term of $O(\log n)$.

Derandomization [BKKK 17], [M16]

For any problem, P, with $I(n)$ possible input sequences of length n

$\exists$ rand. c-comp. alg. for P $\Rightarrow$ $b_P(n,c) \in O(\log\log I(n) + \log n)$

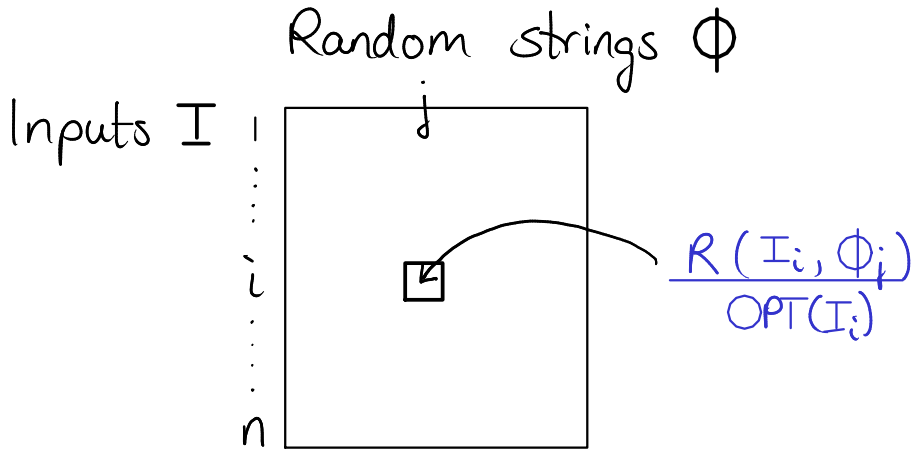
$\therefore$ For Paging, $b(n, H_k) \in O(\log n)$

[BKKK 17]:
Böckenhauer, Komm, Královič, Královič
On the advice complexity of the k-Server problem
Journal of Computer and System Sciences 2017

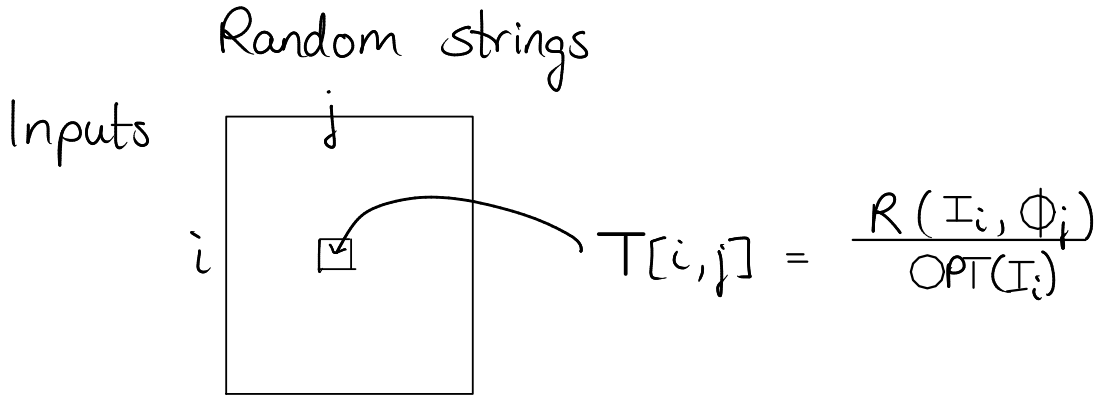
[M16]:
Jesper W Mikkelsen
Randomization can be as helpful as a glimpse of the future in online computation
ICALP 2016

Derandomization of c-competitive randomized alg. R: [BKKK 17]

Using a table:



Obtaining a $c\cdot(1+\varepsilon)$-competitive deterministic algorithm using $O\left(\log \frac{\log I(n)}{\log(1+\varepsilon)} + \log n\right)$ advice bits.

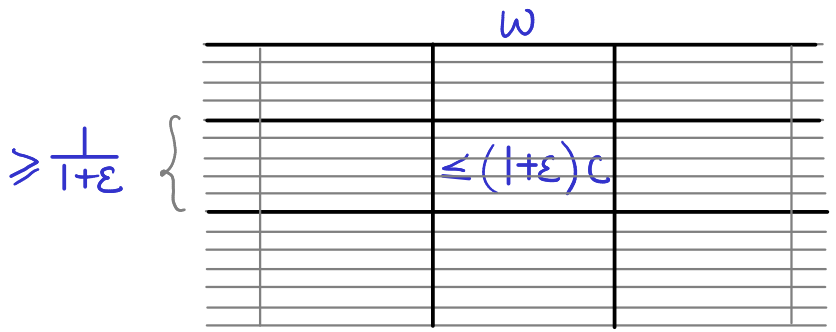


Invariant: $\forall$ row i : $E_j[A[i,j]] \le c \quad \Rightarrow$

$\exists$ column w : $E_i[A[i,w]] \le c$

In column w, $\ge \frac{1}{1+\varepsilon}$ of the entries are $\le (1+\varepsilon)c$ (Markov's Inequality)

The corresponding inputs are assigned w as advice string.



$\le \ell$ strings, where $(1+\varepsilon)^\ell \ge I(n) \quad \Leftrightarrow \quad \ell \le \frac{\log I(n)}{\log(1+\varepsilon)}$

[BKKKM 17]:

Even a few advice bits may improve the competitive ratio:

b advice bits $\Rightarrow$ $CR \leq \frac{2k+2}{2^b} + 3b$ (average of 2^b algorithms)

$\log k$ advice bits $\Rightarrow$ $C.R. \in O(\log k)$

On the other hand:

b advice bits $\Rightarrow$ $CR \geq \frac{k}{2^b}$

Follows from:

- $\forall I$ with $k+1$ different pages : $OPT(I) \leq \frac{n}{k}$
- $\forall A : \exists I$ with $k+1$ different pages : $A(I) \geq \frac{n}{2^b}$

  By induction on i:

  $\leq i$ different advice strings $\Rightarrow$

  $\exists I$ with $k+1$ diff. pages s.t. $|I| = \frac{n}{2^b} \cdot i$ and $ALG(I) \geq \frac{n}{2^b}$

[M16]:

Without advice, best randomized c.r. $= H_k$

$\Rightarrow$ $b(n, H_k - \varepsilon) \in \Omega(n)$ (even for randomized algorithms)

since Paging is Σ-repeatable and compact

$$A(\sigma_1 \sigma_2 \cdots \sigma_r) = \sum_{i=1}^{r} A(\sigma_i) + O(r)$$

Lower bound can be proven via input distribution with finite support

More generally, MTS is Σ-repeatable and compact.

Hence, for any problem that can be modeled as a Metrical Task System,

any lower bound for randomized algorithms without advice is a lower bound for randomized algorithms with $O(n)$ advice bits.

Similarly, deterministic lower bounds carry over to (randomized) algorithms with $o(n)$ advice bits, if
- the bound holds even if OPT and n are known to the alg. and
- the problem is V-repeatable.

$$A(\sigma_1 \sigma_2 \cdots \sigma_r) = \max_{1 \le i \le r} A(\sigma_i)$$

Paging with Predictions [LV18]

For each request,
predict when the same page will be requested again.

Combine advantages of ML with worst-case guarantees:

- If predictions are good,
  the algorithm should be close to optimal (consistency)
- If predictions are bad,
  the algorithm should not be too much worse than
  the best online algorithm (robustness)

[LV18]:
Lykouris, Vassilvitski
Competitive Caching with Machine Learned Advice
ICML 18 and JACM 21

For each request,
predict when the same page will be requested again.

Error measure : ℓ_1

Ex :

	1	2	3	4	5	6	7	8	9	10	11	12	13
I :	1	2	1	3	4	1	3	4	1	2	1	3	2
Prediction :	4	8	6	7	8	10	9	15	11	13	14	14	16
Actual :	3	10	6	7	8	9	12	14	11	13	14	14	14
Error :	1	2	0	0	0	1	3	1	0	0	0	0	2

Total error : $\eta = 1 + 2 + 1 + 3 + 1 + 2 = 10$

$$\varepsilon = \frac{\eta}{OPT} = \frac{10}{2} = 5$$

↑ if $k = 3$

Marking alg (Class of alg.)

For each request
  Mark requested page
  If fault
    If all pages in cache are marked
      Remove all marks (Current req. starts a new k-phase)
    Evict an unmarked page

Ex: LRU: Least Recently Used

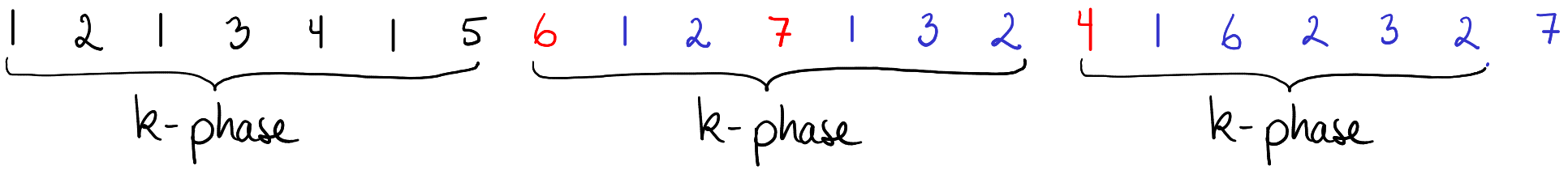
$CR = k$

Mark: Evict random unmarked page

$CR = 2H_k - 1$, where

$$H_k = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k} \approx \ln k$$

Ex: $k=5$

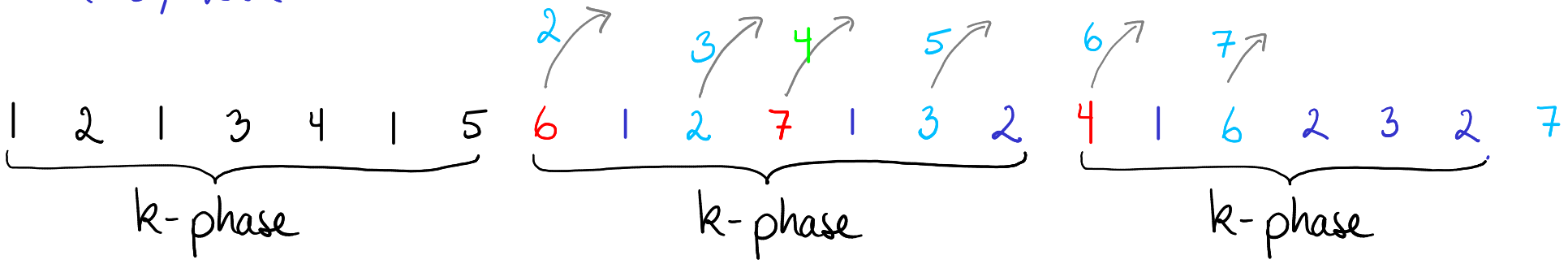


k-phase: Maximal subsequence with k distinct pages

New page: Not req. in previous phase
Old page: Requested in previous phase

Note:
$OPT \geq \frac{N}{2}$, where N = total # new pages

Ex: $k=5$, LRU



Two chains:
$6 \to 2 \to 3 \to 5$
$7 \to 4$

One chain:
$4 \to 6 \to 7$

Note: A chain always starts with a new page.
($\Rightarrow$ #chains $\leq 2 \cdot OPT$)

Note: total #faults = total chain length

For Mark (evicting random unmarked pages),
the expected length of a chain is $\leq H_k \approx \ln k$

Predictive Marker [LV18]

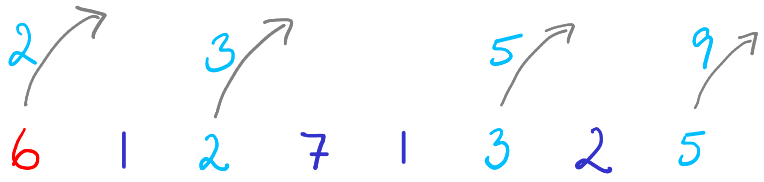
On a fault

...

LFD_p on unmarked pages

...

Ex:



Chain: $6 \to 2 \to 3 \to 5 \to 9$

The pages of the chain are predicted to arrive in the relative order 9, 5, 3, 2 (by the def. of LFD_p)

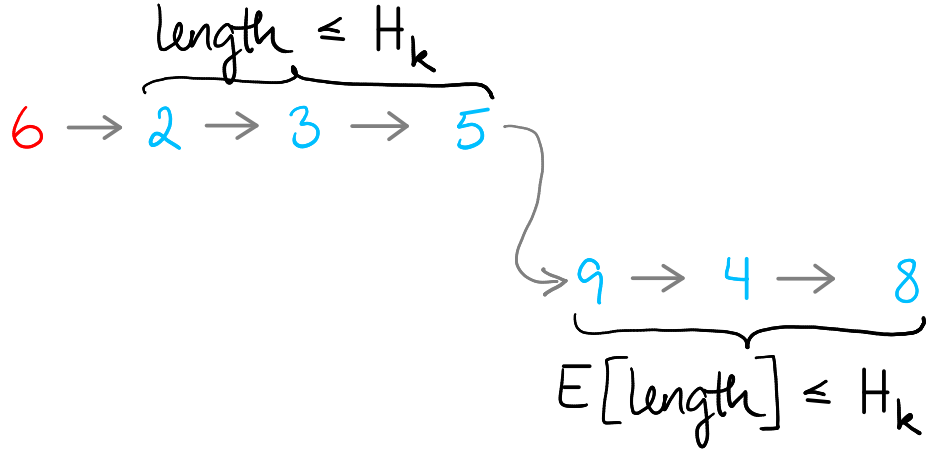
$\Rightarrow$ #inversions $\geq 3+2+1 = \frac{4 \cdot 3}{2}$ $\Rightarrow$ $\eta \geq \frac{4 \cdot 3}{2}$

A chain of length ℓ contributes $\Omega(\ell^2)$ to η.

Predictive Marker is $(2 + O(\sqrt{\varepsilon}))$ - consistent

Predictive Marker [LV18]

On a fault
- If chain shorter than H_k
  - LFD_P on unmarked pages
- Else
  - Evict random unmarked page



Predictive Marker is $4H_k$-robust

$$C.R.(P.M.) \leq \min\{\underbrace{2+4\sqrt{5\epsilon}}_{\text{worst-case}}, \underbrace{4H_k}_{\text{expected}}\}$$

Predictive Marker [LV18]

On a fault
  If chain shorter than H_k
    LFD_p on unmarked pages
  Else
    Evict random unmarked page

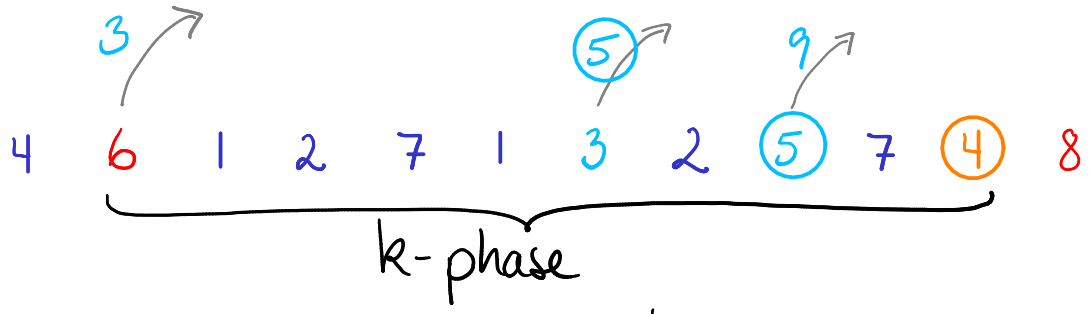
$$C.R.(P.M.) \leq 2 \min\{1 + 2\sqrt{5\varepsilon}, 2H_k\}$$

„Modified Predictive Marker"

On a fault
  If the requested page is new
    LFD_p on unmarked pages
  Else
    Evict random unmarked page

$$C.R.(MPM) \leq 2 \cdot (2 + \min\{H_{2\varepsilon}, H_k\})$$

[R20]:
Rohatgi
Near-Optimal Bounds for
Online Caching with
Machine Learned Advice
SODA 20



4 is predicted to arrive before 5

m unmarked pages ⇒ m inversions

„Modified Predictive Marker" [R20]

On a fault
  If the requested page is new
    LFD_p on unmarked pages
  Else
    Evict random unmarked page

6 → 3 → 5 → 9

E[length] ≤ H_m

$$C.R.(MPM) \leq 2 \cdot (2 + \min\{H_{2\varepsilon}, H_k\})$$

[W20]:

$$C.R.(LFD_p) \leq \min\left\{1+2\varepsilon,\ 4+\frac{4\varepsilon}{k-1}\right\}$$

Combine LFD_p with LRU via a deterministic „blackbox" [FKLMSY91]

$$Combine_D(LFD_p, LRU)(I) \leq 2 \cdot \min\{LFD_p(I), LRU(I)\} \quad \text{for any } I$$

Combine LFD_p with Equitable via a randomized „blackbox" [BB00]

$$Combine_R(LFD_p, Eq, \gamma)(I) \leq (1+\gamma)\cdot\min\{LFD_p(I), Eq(I)\} + O\left(\frac{1}{\gamma}\right)$$

for any I and $0 < \gamma < \frac{1}{4}$

[W20]: Wei
Better and Simpler Learning-Augmented Online Caching
APPROX 20

[FKLMSY91]: Fiat, Karp, Luby, McGeoch, Sleator, Young
Competitive Paging Algorithms
J. Alg. 91

[BB00]: Blum, Burch
On-Line Learning and the Metrical Task Systems Problem
Mach. Learn. 00

[W20]:

$$C.R.(LFD_p) \leq \min\left\{1+2\varepsilon,\ 4+\frac{4\varepsilon}{k-1}\right\}$$

COMBINE$_D$ (LFD_p, LRU)

Simulate LFD_p and LRU
On every second fault
  Evict page $\notin$ LFD_p's cache after serving the request
On every second fault
  Evict page $\notin$ LRU's cache after serving the request

Or

Follow-the-Leader (LFD_p, LRU)

$$C.R.(\text{Combine}(LFD_p, LRU)) \leq \min\left\{2+4\varepsilon,\ 8+\frac{8\varepsilon}{k-1},\ 2k\right\}$$

For any deterministic alg. A, $C.R.(A) \in \Omega\left(1+\frac{\varepsilon}{k}\right)$

For any deterministic alg. A, $C.R.(A) \in \Omega\left(1+\frac{\varepsilon}{k}\right)$

Sketch:

$2 \leq j \leq k-1$

Repeat:

$$\langle p_1^k, p_2^k, \ldots, p_k^k \rangle^k \langle p_{k+1} \rangle \langle ? \rangle^j \langle p_{k+1} \rangle^{k^2+1}$$

just evicted ↑

$$\left.\begin{array}{l} \eta \in O(jk) \\ OPT = 2 \end{array}\right\} \Rightarrow \varepsilon \in O(jk) \Rightarrow j \in \Omega\left(\frac{\varepsilon}{k}\right)$$

$$ALG \geq j+1 = OPT + j-1 = \left(1 + \frac{j-1}{2}\right) OPT = \left(1 + \Omega\left(\frac{\varepsilon}{k}\right)\right) OPT$$

$$> \left(1 + \frac{j}{4}\right) OPT, \quad \text{since } j \geq 2$$

$$= \left(1 + \Omega\left(\frac{\varepsilon}{k}\right)\right) OPT$$

[W20]:

$\underline{\text{Combine}_R}(\text{LFD}_p, \text{Equitable}, 0<\gamma<\frac{1}{4})$ (Randomized Weighted Majority)

$\beta \leftarrow 1-\frac{\gamma}{2}$

$w_{LFD} \leftarrow 1$, $w_{EQ} \leftarrow 1$

On each request

For $A \in \{\text{LFD}_p, \text{EQUITABLE}\}$

If A faults

$w_A \leftarrow \beta \cdot w_A$

On each fault

$p_{\text{LFD}_p} \leftarrow \frac{w_{\text{LFD}_p}}{w_{\text{LFD}_p}+w_{EQ}}$, $p_{EQ} \leftarrow \frac{w_{EQ}}{w_{\text{LFD}_p}+w_{EQ}}$

Choose algorithm using probabilities p_{LFD_p} and p_{EQ}

$C.R.(\text{Combine}_R(\text{LFD}_p, \text{Equitable}, \gamma)) \leq (1+\gamma)\min\{1+2\varepsilon, 4+\frac{4\varepsilon}{k-1}, H_k\}$, $\gamma>0$

Unit Profit Knapsack: Maximize #items packed

Threshold Algorithm [BFK22]

Advice: average item size, a, in OPT

$$T(i) = \frac{ae}{aei+1}$$

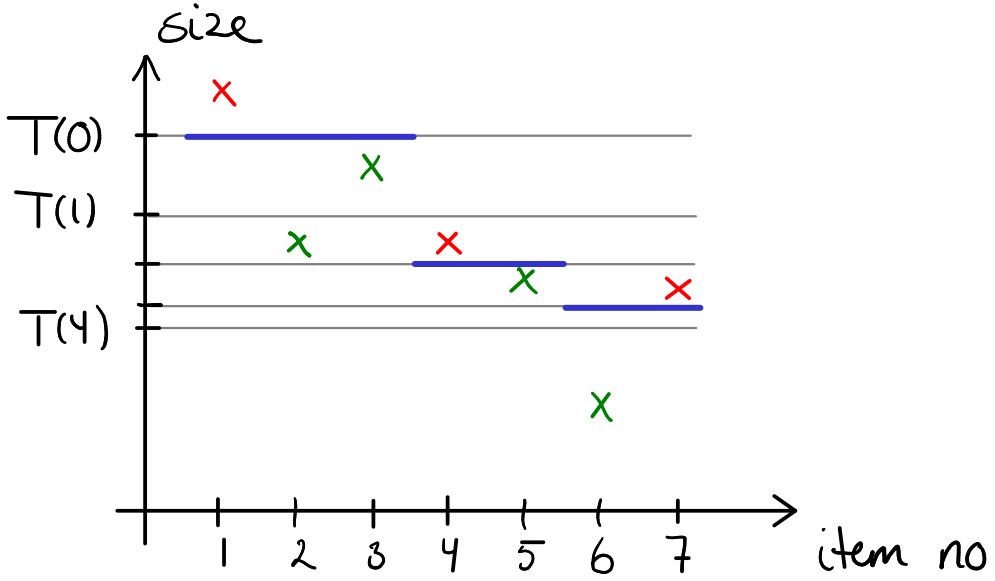
$t = \min\{ T(i) \mid i \text{ items} \geq T(i) \text{ in knapsack} \}$

If current item fits in knapsack and has size $\leq t$

Accept

$\frac{e}{e-1}$ - competitive

Optimal among alg. knowing only a



[BFK22]:
Boyar, Favrholdt, Larsen
Online Unit Profit Knapsack with Untrusted Predictions
SWAT 2022

Unit Profit Knapsack: Maximize #items packed

Average item size in OPT → $a = r \cdot \hat{a}$ ← predicted value of a

Threshold Algorithm [BFK22]

Prediction: average item size, $\hat{a}$, in OPT

$$T(i) = \sqrt{\frac{\hat{a}}{2i}}$$

$$t = \min\{ T(i) \mid i \text{ items} \geq T(i) \text{ in knapsack} \}$$

If current item fits in knapsack and has size $\leq t$

Accept

$$\text{c.r.} = \begin{cases} 2r, & r \geq 1 \\ \frac{2}{r}, & r \leq 1 \end{cases}$$

No alg. knowing only $\hat{a}$ can be better for both $r > 1$ and $r < 1$.

Bin Packing

$1.54 < C.R. < 1.58$

Advice: m = # medium sized items $\left(\frac{1}{2} < \text{size} \leq \frac{2}{3}\right)$

$C.R. = 1.5$ [BKLL-O 16]

Advice: k-bit approx. of $\frac{m}{m+t}$, t = # bins used for tiny items

$C.R. \approx 1.47$ [ADKRR 18]

[BKLL-O 16]: Boyar, Kamali, Larsen, Lopez-Ortiz
Online Bin Packing with Advice
Algorithmica 16

[ADKRR 18]: Angelopoulos, Dürr, Kamali, Renault, Rosén
Online Bin Packing with Advice of Small Size
ToCS 18

Bin Packing

Advice: m = # medium sized items $\left(\frac{1}{2} < \text{size} \leq \frac{2}{3}\right)$

Advice: k-bit approx. of $\frac{m}{m+t}$, t = # bins used for tiny items

Prediction: k-bit approx. of $\frac{m}{m+t}$ [ADJKR 20]

$\alpha \in [0,1]$ is a parameter of trust

$(1.5 + f_c(\alpha, k))$ - consistent (decreasing in α)

$(1.5 + f_r(\alpha))$ - robust (increasing in α)

[ADJKR 20]: Angelopoulos, Dürr, Jin, Kamali, Renault
Online Computation with Untrusted Advice
ITCS 20

Summing up

Advice Complexity

How much extra information does it take to
- be (offline) optimal ?
- beat best online algorithm?

Algorithms may be stepping stones to
- semi-online algorithms
- parallel solutions
- algorithms with predictions

Lower bounds give impossibility results for
- randomized online algorithms (sometimes)
- semi-online algorithms
- parallel solutions

(Online) algorithms with predictions

- New and rapidly evolving field
- Some keywords: Predictions, error measures, consistency, robustness

Summing up

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THANK YOU !!

(Online) algorithms with predictions

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